Manipulation of Simplified Stress-Strain Matrices

V Casey, 2017

Note: apart from the commas between suffixes, the notation is the same as that presented in the 'Simplified Matrix' section of the notes.

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> restart;
> with(LinearAlgebra):
>
> Stress := Matrix(6, 1, symbol=σ);
```

$$Stress := \begin{bmatrix} \sigma_{1,1} \\ \sigma_{2,1} \\ \sigma_{3,1} \\ \sigma_{4,1} \\ \sigma_{5,1} \\ \sigma_{6,1} \end{bmatrix} \quad (1)$$

```
> Strain := Matrix(6, 1, symbol=ε);
```

$$Strain := \begin{bmatrix} \epsilon_{1,1} \\ \epsilon_{2,1} \\ \epsilon_{3,1} \\ \epsilon_{4,1} \\ \epsilon_{5,1} \\ \epsilon_{6,1} \end{bmatrix} \quad (2)$$

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> Stiffness := Matrix(6, 6, symbol=C);
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$$Stiffness := \begin{bmatrix} C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} & C_{1,5} & C_{1,6} \\ C_{2,1} & C_{2,2} & C_{2,3} & C_{2,4} & C_{2,5} & C_{2,6} \\ C_{3,1} & C_{3,2} & C_{3,3} & C_{3,4} & C_{3,5} & C_{3,6} \\ C_{4,1} & C_{4,2} & C_{4,3} & C_{4,4} & C_{4,5} & C_{4,6} \\ C_{5,1} & C_{5,2} & C_{5,3} & C_{5,4} & C_{5,5} & C_{5,6} \\ C_{6,1} & C_{6,2} & C_{6,3} & C_{6,4} & C_{6,5} & C_{6,6} \end{bmatrix} \quad (3)$$

```
> STRESS = MatrixMatrixMultiply(Stiffness, Strain);
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$$STRESS = \begin{bmatrix} C_{1,1}\epsilon_{1,1}+C_{1,2}\epsilon_{2,1}+C_{1,3}\epsilon_{3,1}+C_{1,4}\epsilon_{4,1}+C_{1,5}\epsilon_{5,1}+C_{1,6}\epsilon_{6,1} \\ C_{2,1}\epsilon_{1,1}+C_{2,2}\epsilon_{2,1}+C_{2,3}\epsilon_{3,1}+C_{2,4}\epsilon_{4,1}+C_{2,5}\epsilon_{5,1}+C_{2,6}\epsilon_{6,1} \\ C_{3,1}\epsilon_{1,1}+C_{3,2}\epsilon_{2,1}+C_{3,3}\epsilon_{3,1}+C_{3,4}\epsilon_{4,1}+C_{3,5}\epsilon_{5,1}+C_{3,6}\epsilon_{6,1} \\ C_{4,1}\epsilon_{1,1}+C_{4,2}\epsilon_{2,1}+C_{4,3}\epsilon_{3,1}+C_{4,4}\epsilon_{4,1}+C_{4,5}\epsilon_{5,1}+C_{4,6}\epsilon_{6,1} \\ C_{5,1}\epsilon_{1,1}+C_{5,2}\epsilon_{2,1}+C_{5,3}\epsilon_{3,1}+C_{5,4}\epsilon_{4,1}+C_{5,5}\epsilon_{5,1}+C_{5,6}\epsilon_{6,1} \\ C_{6,1}\epsilon_{1,1}+C_{6,2}\epsilon_{2,1}+C_{6,3}\epsilon_{3,1}+C_{6,4}\epsilon_{4,1}+C_{6,5}\epsilon_{5,1}+C_{6,6}\epsilon_{6,1} \end{bmatrix} \qquad \textbf{(4)}$$

Cubic Crystal - three constants.

> $Stiffness := \langle\langle C_{1,1}, C_{1,2}, C_{1,2}, 0, 0, 0\rangle | \langle C_{1,2}, C_{1,1}, C_{1,2}, 0, 0, 0\rangle | \langle C_{1,2}, C_{1,2}, C_{1,1}, 0, 0, 0\rangle | \langle 0, 0, 0, C_{4,4}, 0, 0\rangle | \langle 0, 0, 0, 0, C_{4,4}, 0\rangle | \langle 0, 0, 0, 0, 0, C_{4,4}\rangle\rangle;$

$$Stiffness := \begin{bmatrix} C_{1,1} & C_{1,2} & C_{1,2} & 0 & 0 & 0 \\ C_{1,2} & C_{1,1} & C_{1,2} & 0 & 0 & 0 \\ C_{1,2} & C_{1,2} & C_{1,1} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{4,4} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{4,4} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{4,4} \end{bmatrix} \qquad \textbf{(2.1)}$$

>

Compliance Matrix

> S:=MatrixInverse(**(2.1)**)

$$S := \left[\left[\frac{C_{1,1}+C_{1,2}}{C_{1,1}^2+C_{1,1}C_{1,2}-2C_{1,2}^2}, -\frac{C_{1,2}}{C_{1,1}^2+C_{1,1}C_{1,2}-2C_{1,2}^2}, -\frac{C_{1,2}}{C_{1,1}^2+C_{1,1}C_{1,2}-2C_{1,2}^2}, 0, 0, 0\right],\right. \qquad \textbf{(3.1)}$$

$$\left[-\frac{C_{1,2}}{(C_{1,1}-C_{1,2})(C_{1,1}+2C_{1,2})}, \frac{C_{1,1}+C_{1,2}}{(C_{1,1}-C_{1,2})(C_{1,1}+2C_{1,2})}, -\frac{C_{1,2}}{(C_{1,1}-C_{1,2})(C_{1,1}+2C_{1,2})}, 0, 0, 0\right],$$

$$\left[-\frac{C_{1,2}}{C_{1,1}^2+C_{1,1}C_{1,2}-2C_{1,2}^2}, -\frac{C_{1,2}}{C_{1,1}^2+C_{1,1}C_{1,2}-2C_{1,2}^2},\right.$$

$$\left.\frac{C_{1,1}+C_{1,2}}{C_{1,1}^2+C_{1,1}\,C_{1,2}-2\,C_{1,2}^2}, 0, 0, 0\right],$$

$$\left[0, 0, 0, \frac{1}{C_{4,4}}, 0, 0\right],$$

$$\left[0, 0, 0, 0, \frac{1}{C_{4,4}}, 0\right],$$

$$\left[0, 0, 0, 0, 0, \frac{1}{C_{4,4}}\right]\right]$$

Individual Elements

> $S_{1,1}$;

$$\frac{C_{1,1}+C_{1,2}}{C_{1,1}^2+C_{1,1}\,C_{1,2}-2\,C_{1,2}^2} \quad \textbf{(4.1)}$$

> $S_{1,2}$;

$$-\frac{C_{1,2}}{C_{1,1}^2+C_{1,1}\,C_{1,2}-2\,C_{1,2}^2} \quad \textbf{(4.2)}$$

> $S_{4,4}$;

$$\frac{1}{C_{4,4}} \quad \textbf{(4.3)}$$

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Inter Relationships

> $C_{1,1} := C_{1,2}+2\,C_{4,4}$;

$$C_{1,1} := C_{1,2}+2\,C_{4,4} \quad \textbf{(5.1)}$$

> $EQN5 := \frac{1}{S_{1,1}}$;

$$EQN5 := \frac{\left(C_{1,2}+2\,C_{4,4}\right)^2+\left(C_{1,2}+2\,C_{4,4}\right)\,C_{1,2}-2\,C_{1,2}^2}{2\,C_{1,2}+2\,C_{4,4}} \quad \textbf{(5.2)}$$

> $algsubs\left(C_{1,2}=\text{lambda}, EQN5\right)$;

$$\frac{\left(\lambda+2\,C_{4,4}\right)^2+\lambda\left(\lambda+2\,C_{4,4}\right)-2\,\lambda^2}{2\,\lambda+2\,C_{4,4}} \quad \textbf{(5.3)}$$

> $algsubs\left(C_{4,4}=\text{mu}, \%\right)$;

(5.4)

$$\frac{(\lambda + 2\,\mu)^2 + \lambda\,(\lambda + 2\,\mu) - 2\,\lambda^2}{2\,\lambda + 2\,\mu} \tag{5.4}$$

> **E:=**$simplify(\ \%, 'assume = positive'\ );$

$$E := \frac{\mu\,(3\,\lambda + 2\,\mu)}{\lambda + \mu} \tag{5.5}$$

> $\text{nu} := -\frac{S_{1,\,2}}{S_{1,\,1}};$

$$\nu := \frac{C_{1,\,2}}{2\,C_{1,\,2} + 2\,C_{4,\,4}} \tag{5.6}$$

> $algsubs(C_{1,\,2} = \text{lambda}, \%);$

$$\frac{\lambda}{2\,\lambda + 2\,C_{4,\,4}} \tag{5.7}$$

> $algsubs(C_{4,\,4} = \mu, \%);$

$$\frac{\lambda}{2\,\lambda + 2\,\mu} \tag{5.8}$$

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